Weak values and quantum-classical correspondence

Dept. Phys., Tokyo Metropolitan Univ. TANAKA Atushi

Aharonov-Albert-Vaidman の弱値を通じた量子古典対応を論じます. まず, 弱値の半古典論から, 古典的に実現し得ない複素古典軌道が自然に現われることを報告します. これは, Ehrenfest 型の量子古典対応の複素軌道への拡張に他なりません. 次いで, 弱い滞在時間としての "トンネル時間" の問題の, 半古典論による解析を報告します. そこでは, potential 障壁に反射される過程で, 古典的に禁じられた領域への侵入では, 不連続性のある古典軌道が現われることを見いだしました.

A classical system is in a definite configuration at any time instant. The series of the configurations during a time interval is called a trajectory, which provides a complete description of the system at the time interval. In contrast with classical theory, the "conventional" (e.g. Copenhagen) interpretation of quantum theory doesn't have the concept of trajectories. Hence it becomes nontrivial (often impossible) to speak about, for example, the configuration of a system at some instant, or, the sojourn of a particle at some region during a time interval (see, e.g. Yamada 1999). However, we may argue that the trajectories have an approximate "reality", at least, within a short time scale, with small quantum fluctuations: the Eherenfest theorem tells us that the expectation values of quantum systems approximately obey the corresponding classical equations of motion. A systematic extension of Ehrenfest-type quantum-classical correspondence to quantum interference phenomena is achieved by the semiclassical theory, where the quantum superpositions of classical trajectories is introduced. In the semiclassical theory, further extension of classical trajectories into *complex-valued* phase-space naturally arises: The complex-valued classical trajectories, which are impossible to "realize" within the classical theory, are employed in semiclassical description of classically-forbidden phenomena, e.g. tunnelings. It is revealed that the complex-valued classical trajectories are indispensable in analysis of classically forbidden, "complex" (e.g. classically chaotic) quantum dynamics, since the complex-valued classical trajectories enable us to incorporate knowledges of complex dynamical systems (see, Shudo et. al 2002 and references therein).

Firstly, I wish to argue about "approximate reality" of the complex-valued classical trajectories, which are obtained through the multiple extensions of the original, classical concept. For example, is it possible to observe complex-valued momentum of particle that is below a potential barrier? Note that the Ehrenfest theorem mentioned above, which concerns only about conventional, real-valued expectation value of quantum ensemble, cannot be applied to this argument. This is the difference from the case with real-valued trajectories. The key to the question above is Aharonov-Albert-Vaidman's weak value, which is an "expectation value" of the quantum ensemble that is specified by both an initially (at $t = t'$)

prepared state $|\psi'\rangle$ and a finally (at $t = t''(> t')$) *postselected* state $|\psi''\rangle$. The weak value of an observable $\hat{A}$ at time $t \in [t', t'']$ is

$$\mathrm{W}[\hat{A}, t] \equiv \frac{\langle \psi'' | \hat{U}(t'', t)\, \hat{A}\, \hat{U}(t, t') | \psi' \rangle}{\langle \psi'' | \hat{U}(t'', t') | \psi' \rangle}$$

where $\hat{U}(t_f, t_i)$ is the time evolution operator during time interval $[t_i, t_f]$ (Aharonov, Albert and Vaidman, 1988). A semiclassical theory of the weak value showed that $\mathrm{W}[\hat{A}, t]$ approximately obeys classical equation of motion, if there exist only a single contribution of a classical trajectory to the semiclassical evaluation of a Feynman kernel $\langle \psi'' | \hat{U}(t'', t') | \psi' \rangle$. Thus we have an extension of Eherenfest-type quantum-classical correspondence of the complex-valued classical trajectories. Note that the present argument doesn't discriminate between real- and complex-valued trajectories (AT 2002).

Secondly, I wish to discuss about multiple-time properties of quantum system, in terms of the trajectory concept. For example, is it possible to determine the tunneling time of a particle below a barrier during a time interval? Here we encounter weak values again. It is natural to choose quantum ensembles with preparation and postselection: e.g. the time evolution of a transmission of wave packet is described by initial and finial wave packets are localized both sides of the potential barrier. The sojourn time T_Ω for the region Ω in configuration space is the integral of the weak value of a projection operator $\hat{P}_\Omega = \int_\Omega |q\rangle\, dq\, \langle q|$,

$$T_\Omega = \int_{t'}^{t''} W[\hat{P}_\Omega, t]\, dt,$$

which is known as Sokolovski and Baskin's tunneling time (1987) (Iannaccone 1996). A semiclassical analysis on tunneling-time problems below a parabolic barrier revealed the following: (1) Both the transmission and reflection processes below the barrier incorporate only a single trajectory. Hence the Ehrenfest-type quantum classical correspondence is possible; (2) The reflecting trajectory experiences a discontinuity of momentum when the trajectory is reflected below the barrier (AT, in preparation).

Finally, a surmise on the "degree of realities" of real- and complex-valued trajectories is given. It is quite plausible that they may be the same: In a short time scale, before Ehrenfest time, they are approximately *realized* as certain expectation values. After Ehrenfest time, these trajectories lose even approximate reality, as is concluded by the founders of quantum theory, due to the breakdown of Ehrenfest-type quantum-classical correspondence.

References

Y. Aharonov, D.Z. Albert and L. Vaidman, Phys. Rev. Lett. **60** (1988), 1351.

G. Iannaccone, quant-ph/9611018 (1996)

A. Shudo et al., J. Phys. A **35** (2002), L225.

D. Sokolovski and L.M. Baskin, Phys. Rev. A **36** (1987) 4604.

AT, Phys. Lett. A **297** (2002), 307; Bussei Kenkyu (Kyoto) **77** (2002), 922.

N. Yamada, Phys. Rev. Lett. **83** (1999), 3350.